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Non-Heisenberg two-dimensional ferromagnet in an external magnetic field

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Abstract. The phase states of a non-Heisenberg two-dimensional ferromagnet are studied, in which the long-range magnetic order is stabilized by the magnetoelastic interaction. It is shown that in this system, together with the phases of nonzero magnetic order, there exists a quadrupolar phase characterized by a tensor order parameter at zero external field. The transition temperature from the quadrupolar phase to the paramagnetic phase is determined.

PACS. 75.10.-b General theory and models of magnetic ordering – 75.30.Kz Magnetic phase boundaries (including magnetic transitions, metamagnetism, etc.)

Introduction

For decades the Heisenberg model has been the basic model on which the theory of magnetism has developed. However, it is also possible to exceed the limits of the bilinear exchange interaction, while not breaking the isotropy of the system. Among the most interesting systems of this class are magnets, in which the exchange Hamiltonian for the higher orders in spin is of the same order of magnitude as the bilinear Heisenberg exchange. It is obvious that more complex models should also be characterized by unusual properties [1–5]. In magnets with a regular crystal lattice magnetic structures were found that are essentially impossible within the Heisenberg model [1,2]. Among them, for example, is the inclined two-sublattice structure (this inclination appears to be huge in comparison with relativistic effects, for example, in the antiferromagnets or in crystals with other symmetry). Another interesting property of such magnets is their magnetic polymorphism. The greatest number of phases (fourteen) was observed in CeBi [1,2]. Although such systems have long been under investigation [1,2,6–8], the interest in them has not decreased. Such studies are important in connection with the synthesis, and the experimental investigation of compounds where the temperature of the magnetic ordering is very low. In such situations, if the spin of a magnetic ion is $S \geq 1$, the Heisenberg exchange interaction is the same order of magnitude, or less than the interaction, which is described by the invariants of higher order [1]. The rare earth pniktids, the cubic intermetallids TmCd, TmZn and also EuO, EuS, EuSe are examples of such systems [1,9–11].

In this context, it is interesting to investigate the spectra of elementary excitations under varying magnetic fields. It is very important because certain singlet magnets exist in the non-magnetic state when the magnetic field $H = 0$, but when the magnetic field H is very large, they are in the magnetic state [1,2,11–14].

Investigations of this kind have been carried out for 3D magnets [1–5,12–17]. It is of interest to study the dynamical properties and phase states of 2D non-Heisenberg magnets. For example, a numerical modeling for EuTe films [18] has been carried out. It should be noted that in such a system the Heisenberg coupling constant exceeds the constant of the biquadratic exchange. However, there is a large number of 2D non-Heisenberg magnets (CrO_2, Mn_2O_3, $CrCl_3$ [19]) in which the constant of the biquadratic exchange is comparable or greater than the constant of the Heisenberg exchange. The aim of the present paper is to study a 2D non-Heisenberg ferromagnet with a large biquadratic exchange interaction constant (exceeding the Heisenberg interaction constant), and spin 1 in an external magnetic field.

Model

Consider a 2D ferromagnet with the biquadratic exchange interaction and a magnetoelastic (ME) interaction in an external magnetic field. The magnetic field H is perpendicular to the plane (XOY), and directed along the axis OZ.

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The Hamiltonian of the system can be written in the form:

$$H = -\frac{1}{2}\sum_{n,n'}\left[J\left(n-n'\right)\vec{S}_n\vec{S}_{n'} + K\left(n-n'\right)\left(\vec{S}_n\vec{S}_{n'}\right)^2\right] - H\sum_n S_n^z + \nu\sum_{n,i,j} S_n^i S_n^j u_{ij} + \int dr\left\{\frac{\lambda+\eta}{2}\left(u_{xx}^2+u_{yy}^2+u_{zz}^2\right) + \eta\left(u_{xy}^2+u_{xz}^2+u_{yz}^2\right) + \lambda\left(u_{xx}u_{yy}+u_{xx}u_{zz}+u_{zz}u_{yy}\right)\right\} \quad (1)$$

where S_n^i is the spin operator at site n; $K\left(n-n'\right) > J\left(n-n'\right) > 0$ are the biquadratic and the Heisenberg interaction constants; respectively, ν is the ME coupling constant; λ, η are the elastic modules; $u_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ is the symmetric part of the components of the elastic strain tensor. From now on we assume that the magnetic ion has spin $S = 1$, since such systems have pronounced quantum properties [1,2].

The magnetoelastic interaction is taken into account because, first, taking it into account makes the model more realistic, and second, it leads to the stabilization of long range magnetic order [22,23]. The long range magnetic order exists in the system when a magnetic field differs from zero $H \neq 0$. However, when there is no magnetic field, the long range magnetic order vanishes in accordance with the Mermin-Wagner theorem [20]. Mathematically it is manifested as the divergence of the fluctuation integral at $k \to 0$.

As was shown in [21–23], the presence of the magnetodipolar, or the ME interaction, leads to either a modification of the spectra of elementary excitations, or to the root dependence of the spectrum on the wave vector in the presence of the magnetodipolar interaction, or to the presence of the magnetoelastic gap in the spectrum. Also it leads to a convergence of the fluctuation integral and thus to the existence of the long-range order.

Separating the exchange part of the Hamiltonian (1) with the mean field value $\langle S\rangle$, which is connected with the ordering of the magnetic moment and the complementary fields (which are connected with the quadrupolar (QU) ordering), we obtain the following one-ion Hamiltonian:

$$H_0(n) = -\tilde{H}S_n^z - B_2^0 Q_{2n}^0 - B_2^2 Q_{2n}^2 + \frac{\nu}{4}\Big\{\left(S_n^+\right)^2\left(u_{xx}-u_{yy}-2iu_{xy}\right) + \left(S_n^-\right)^2\left(u_{xx}-u_{yy}+2iu_{xy}\right) + 4u_{zz}\left(S_n^z\right)^2 + \left(u_{xx}+u_{yy}\right)\left(S_n^+S_n^- + S_n^-S_n^+\right) + 2u_z^-\left(S_n^+S_n^z + S_n^zS_n^+\right) + 2u_z^+\left(S_n^-S_n^z + S_n^zS_n^-\right)\Big\}, \quad (2)$$

where

$$\tilde{H} = H + \left(J_0 - \frac{K_0}{2}\right)\langle S^z\rangle,$$
$$u_z^{\pm} = u_{zx} \pm iu_{yz},$$
$$B_2^0 = \frac{K_0}{6}q_2^0,$$
$$B_2^2 = \frac{K_0}{2}q_2^2,$$
$$q_2^p = \langle Q_2^p\rangle,\ (p = 0,\ 2,\ xy,\ zx,\ xy),$$
$$J_0 = \sum_{n'} J(n-n'),\ K_0 = \sum_{n'} K(n-n'),$$
$$Q_{2n}^0 = 3\left(S_n^z\right)^2 - S\left(S+1\right),$$
$$Q_{2n}^2 = \frac{1}{2}\left[\left(S_n^+\right)^2 + \left(S_n^-\right)^2\right],\ Q_{2n}^{xy} = S_n^x S_n^y + S_n^y S_n^x,\ \ldots$$

We have taken into account the fact that due to the symmetry of the problem:

$$q_2^{xy} = q_2^{xz} = q_2^{zy} = 0.$$

As follows from (2), the presence of the QU field results in the appearance an effective anisotropy with the constants B_2^0 and B_2^2.

Solving the Schrödinger equation with the Hamiltonian (2), we obtain the following energy levels of the magnetic ion:

$$E_1 = -3B_2^0 + \frac{\nu}{2}\left(u_{xx}+u_{yy}+2u_{zz}\right) - \frac{\chi}{2},$$
$$E_0 = \nu\left(u_{xx}+u_{yy}\right),$$
$$E_{-1} = -3B_2^0 + \frac{\nu}{2}\left(u_{xx}+u_{yy}+2u_{zz}\right) + \frac{\chi}{2}, \quad (3)$$

where $\chi^2 = 4\tilde{H}^2 + 4\nu^2 u_{xy}^2 + [2B_2^2 - \nu(u_{xx}+u_{yy})]^2$, and the eigenfunctions of the Hamiltonian (2):

$$\Psi\left(1\right) = \cos\theta\left|1\right\rangle + \sin\theta\left|-1\right\rangle,$$
$$\Psi\left(0\right) = \left|0\right\rangle,\ \Psi\left(-1\right) = -\sin\theta\left|1\right\rangle + \cos\theta\left|-1\right\rangle. \quad (4)$$

Here

$$\cos\theta = \sqrt{\frac{\sqrt{\tilde{H}^2 + \left(B_2^2\right)^2} + \tilde{H}}{2\sqrt{\tilde{H}^2 + \left(B_2^2\right)^2}}},$$

$$\sin\theta = \sqrt{\frac{\sqrt{\tilde{H}^2 + \left(B_2^2\right)^2} - \tilde{H}}{2\sqrt{\tilde{H}^2 + \left(B_2^2\right)^2}}}.$$

The spontaneous strains u_{ij} in (3) are determined from the free energy density minimum and have the form:

$$u_{xx}^{(0)} = \frac{\nu}{2\eta(\eta+3\lambda)} \times \frac{(\lambda-\eta)\, ch\frac{\chi_0}{T} + (\eta+3\lambda)\frac{B_2^2}{\chi_0} sh\frac{\chi_0}{T} - (\lambda+\eta)\, e^{-3B_2^0/T}}{2ch\frac{\chi_0}{T} + e^{-3B_2^0/T}},$$

$$u_{yy}^{(0)} = \frac{\nu}{2\eta(\eta+3\lambda)} \times \frac{(\lambda-\eta)\, ch\frac{\chi_0}{T} - (\eta+3\lambda)\frac{B_2^2}{\chi_0} sh\frac{\chi_0}{T} - (\lambda+\eta)\, e^{-3B_2^0/T}}{2ch\frac{\chi_0}{T} + e^{-3B_2^0/T}},$$

$$u_{zz}^{(0)} = -\frac{2\nu}{\eta(\eta+3\lambda)} \frac{(\lambda+\eta)\, ch\frac{\chi_0}{T} - \lambda e^{-3B_2^0/T}}{2ch\frac{\chi_0}{T} + e^{-3B_2^0/T}},$$

$$u_{xy}^{(0)} = u_{xz}^{(0)} = u_{yz}^{(0)} = 0,$$

$$\chi_0 = \sqrt{\tilde{H}^2 + \left(B_2^2\right)^2}. \tag{5}$$

In the basis of the eigenfunctions of the one-ion Hamiltonian, we construct the Hubbard operators $X_n^{M'M} \equiv |\Psi_n(M')\rangle\langle\Psi_n(M)|$ [24–27], which describe the transitions of the magnetic ion from the state M' to the state M. In this case they are related to the spin operators through:

$$S_n^+ = \sqrt{2}\left(\sin\theta\left(X_n^{01} - X_n^{-10}\right) + \cos\theta\left(X_n^{0-1} + X_n^{10}\right)\right);$$

$$S_n^- = \sqrt{2}\left(\sin\theta\left(X_n^{10} - X_n^{0-1}\right) + \cos\theta\left(X_n^{-10} + X_n^{01}\right)\right);$$

$$S_n^z = \cos 2\theta\left(X_n^{11} - X_n^{-1-1}\right) - \sin 2\theta\left(X_n^{1-1} + X_n^{-11}\right). \tag{6}$$

Using this connection, we determine the order parameters of the system:

$$\langle S^z\rangle = \cos 2\theta \frac{e^{-E_1/T} - e^{-E_{-1}/T}}{e^{-E_1/T} + e^{-E_0/T} + e^{-E_{-1}/T}},$$

$$q_2^0 = 3\frac{e^{-E_1/T} + e^{-E_{-1}/T}}{e^{-E_1/T} + e^{-E_0/T} + e^{-E_{-1}/T}} - 2,$$

$$q_2^2 = \sin 2\theta \frac{e^{-E_1/T} - e^{-E_{-1}/T}}{e^{-E_1/T} + e^{-E_0/T} + e^{-E_{-1}/T}}. \tag{7}$$

In the case of low temperatures (at $H \neq 0$), the order parameters have the following form:

$$\langle S^z\rangle = \cos 2\theta,\ q_2^0 = 1,\ q_2^2 = \sin 2\theta, \tag{8}$$

where we have taken into account that the lowest energy level is E_1 (at $H \neq 0$).

Thus, if the magnetic field is greater than some "critical" value, the ferromagnetic (FM) phase is implemented in our system; in this case $\langle S^z\rangle = 1$, $q_2^0 = 1$, $q_2^2 = 0$. The "critical" field is determined from the spectra of quasiparticles.

The phase diagram of a non-Heisenberg ferromagnet in an external magnetic field

The spectra of quasiparticles is determined by the poles of Green's function [28]:

$$G^{\alpha\alpha'}(n, n'; \tau, \tau)' = -\left\langle \hat{T}\tilde{X}_n^{\alpha}(\tau)\, \tilde{X}_{n'}^{\alpha'}(\tau')\right\rangle$$

where $\hat{T}$ is the Wick operator, $\tilde{X}_n^{\alpha}(\tau)$ is the Hubbard operator in the Heisenberg representation, and the averaging is carried out with the total Hamiltonian $H = H_{int} + H_{tr} + H_0$, where H_{int} is the exchange Hamiltonian, H_0 is the one-ion Hamiltonian, and H_{tr} is the Hamiltonian of magnon-phonon transformation [5] which has the following explicit form:

$$H_{tr} = \frac{1}{\sqrt{N}} \sum_{n,\alpha,q,\lambda} \left(b_{q,\lambda} + b_{-q,\lambda}^{+}\right) T_n^{\alpha}(q,\lambda) X_n^{\alpha}.$$

Here, $T_n^{\alpha}(q,\lambda)$ are the amplitudes of magnon-phonon transformations, the square of which determine the probability of the corresponding transition, while $b_{-q,\lambda}^{+}$, $b_{q,\lambda}$ are the creation and annihilation operators of λ-polarized magnons, respectively. Since the magnetoelastic coupling leads to the hybridization of magnetic and elastic excitations, there exist quasimagnon and quasiphonon excitation rather than just magnons and phonons in the system [3,31].

Details of the procedure for obtaining the dispersion equation to describe the spectra of coupled ME waves are given in [5]. Since this equation has a rather cumbersome form, we do not provide it here, and we restrict ourselves to the analysis of its solutions for the case $\vec{k} \parallel OY$. For such geometry, the only nonzero components of a unit polarization vector of a phonon are e_l^y, e_t^z, e_τ^x. This equation is valid for all values of material constants, therefore it permits us to determine the spectra of coupled ME waves in various phases. We now consider this equation in the quadrupolar and ferromagnetic phases.

1 Ferromagnetic phase

Let us first consider the case where the magnetic field is large while the temperature is relatively small. In this case, the FM phase occurs in the system, and the spectra of the "transverse" and "longitudinal" quasimagnons have the following respective forms:

$$\varepsilon_{\perp}(k) = \frac{J_0}{2}k^2 + H + a_0,$$

$$\varepsilon_{||}(k) = \frac{K_0}{2}k^2 + 2\left[H - K_0 + J_0\right].$$

The spectra of $\tau-$polarized quasiphonons are determined by the following expression:

$$\omega^2(k) = \omega_\tau^2(k)\, \frac{\frac{K_0}{2}k^2 + 2\left[H - K_0 + J_0 - \frac{a_0}{2}\right]}{\frac{K_0}{2}k^2 + 2\left[H - K_0 + J_0\right]} \tag{9}$$

where $a_0 = \frac{\nu^2}{2\eta}$ is the parameter of the ME coupling. We assume that the lattice parameter equals unity.

The spectrum of $t-$ and $l-$polarized sound waves remain linear in the wave vector. This demonstrates that they do not interact with the magnetic subsystem.

Following the expression (9), at $H = H_c = K_0 - J_0 + \frac{a_0}{2}$ the spectrum of τ-polarized quasiphonons softens. In the case of the long wavelength limit it is given by:

$$\omega^2(k) = \omega_\tau^2(k)\frac{K_0 k^2}{2a_0},$$

while the spectrum of "longitudinal" quasimagnons exhibits a ME gap $\varepsilon_{||}(0) = a_0$. It should be noted that in the FM phase $\langle (S^z)^2\rangle = 1$, $\langle (S^x)^2\rangle = \langle (S^y)^2\rangle = 1/2$.

Thus, H_c is the field of the phase transition from the FM phase (with the order parameters determined in (8)) to the quadrupolar-ferromagnetic (QFM) (or the angular) phase. In this phase, the order parameters are functions of the field, and:

$$\langle S^z\rangle = \cos 2\theta < 1,\ q_2^2 \approx \sin 2\theta \neq 0,\ q_2^0 < 1.$$

This phase exists for $0 < H < H_c$.

2 Quadrupolar phase

It is interesting to study the case where $H = 0$. In this case, as follows from (3), E_0 is the lowest energy level and the order parameters determined by (7) (at $H = 0$) can be presented in the following form:

$$\langle S^z\rangle = 0,\ q_2^2 = 0,\ q_2^0 = -2. \tag{10}$$

These order parameters (10) are determined in the QU phase [1–3,6–8,15,16]. It is necessary to take into consideration that $\cos 2\theta = 0$ in the QU phase, and that $\sin 2\theta = 1(\sin\theta = \cos\theta = \frac{1}{\sqrt{2}})$, however $q_2^2 = 0$ and $\langle (S^z)^2\rangle = 0$ because of the inversion of the energy levels. Besides, in this phase: $\langle (S^x)^2\rangle = \langle (S^y)^2\rangle = 1$.

Let us investigate the spectra of elementary excitations in the QU phase. We use the method of bosonization of the Hubbard operators [29,30]. We associate the operators X_n^α with pseudo-Hubbard operators $\tilde{X}_n^\alpha$, operating in the Hilbert space and connected with Bose creation and annihilation operators of the quasiparticles (magnons) through:

$$\begin{aligned}
\tilde{X}_n^{01} &= (1 - a_n^+ a_n - b_n^+ b_n)a_n,\\
\tilde{X}_n^{10} &= a_n^+,\\
\tilde{X}_n^{-11} &= b_n^+ a_n,\\
\tilde{H}_n^{1} &= a_n^+ a_n,\\
\tilde{H}_n^{0} &= 1 - a_n^+ a_n - b_n^+ b_n\\
\tilde{X}_n^{0-1} &= (1 - a_n^+ a_n - b_n^+ b_n)b_n,\\
\tilde{X}_n^{-10} &= b_n^+,\\
\tilde{X}_n^{1-1} &= a_n^+ b_n,\\
\tilde{H}_n^{-1} &= b_n^+ b_n.
\end{aligned} \tag{11}$$

Using (11), the Hamiltonian of our system (in the QU phase) can be recast through Bose-operators:

$$\begin{aligned}
H = \sum_k a_k^+ a_k\left[E_{10} - J(k)\right] &+ \frac{1}{2}\sum_k\left(a_k^+ a_{-k}^+ + a_k a_{-k}\right)\left[K(k) - J(k)\right]\\
&+ \sum_k b_k^+ b_k\left[E_{-10} - J(k)\right]\\
&+ \frac{1}{2}\sum_k\left(b_k^+ b_{-k}^+ + b_k b_{-k}\right)\left[K(k) - J(k)\right]
\end{aligned} \tag{12}$$

where $a^+(a)$ are the Bose creation (annihilation) operators of low-frequency magnons, $b^+(b)$ are the Bose creation (annihilation) operators of high-frequency magnons, and $E_{ij} = E_i - E_j$, where E_i is the energy level of the magnetic ion ($i = 1, 0, -1$).

Using Bogolyubov's u-ν transformation [31], it is straightforward to diagonalize the Hamiltonian (12):

$$H = \sum_k \varepsilon_\alpha(k)\alpha_k^+\alpha_k + \sum_k \varepsilon_\beta(k)\beta_k^+\beta_k, \tag{13}$$

where $\varepsilon_\alpha^2(k) = \left[E_{10} - J(k)\right]^2 - \left[K(k) - J(k)\right]^2$, $\varepsilon_\beta^2(k) = \left[E_{-10} - J(k)\right]^2 - \left[K(k) - J(k)\right]^2$.

It is necessary to note that given equation (5) in the QU phase, the spontaneous deformations have the form $u_{xx} = u_{yy} \approx -\frac{\nu}{\eta}$, which is why there is the degeneracy of energy levels ($E_1 = E_{-1}$) in the QU phase, leading to the equality $\varepsilon_\alpha = \varepsilon_\beta$.

Taking equation (3) into consideration, the spectrum of magnons can be presented in the following form:

$$\varepsilon^2(k) = \left[a_0 + \frac{K_0}{2}k^2\right]\left[a_0 + 2K_0 - 2J_0\right]. \tag{14}$$

Since we are searching for the existence of the long-range magnetic order in the system, we consider the behavior of the order parameter q_2^0 which can be presented in the following form:

$$q_2^0 = \frac{3}{N}\sum_n\left\langle (S_n^z)^2\right\rangle - 2 = \frac{1}{N}\sum_n\left\langle 1 - 3a_n^+ a_n\right\rangle. \tag{15}$$

In obtaining equation (15), we use the fact that in the QU phase, $\langle a_n^+ a_n\rangle = \langle b_n^+ b_n\rangle$, and the magnetic ion has spin $S = 1$.

Equation (15) then yields:

$$q_2^0 = 1 - \frac{3}{(2\pi)^2}\int_0^\infty\left(u_k^2 + \nu_k^2\right)\frac{kdk}{\exp\left(\frac{\varepsilon(k)}{T}\right) - 1} + q(0), \tag{16}$$

where u_k and ν_k are the above determined Bogolyubov parameters, $u_k^2 + \nu_k^2 = \frac{E_{10} - J(k)}{\varepsilon(k)}$, and $q(0) = \frac{1}{(2\pi)^2}\int_0^\infty \nu_k^2\frac{kdk}{\exp(\frac{\varepsilon(k)}{T}) - 1}$ are the zero oscillations.

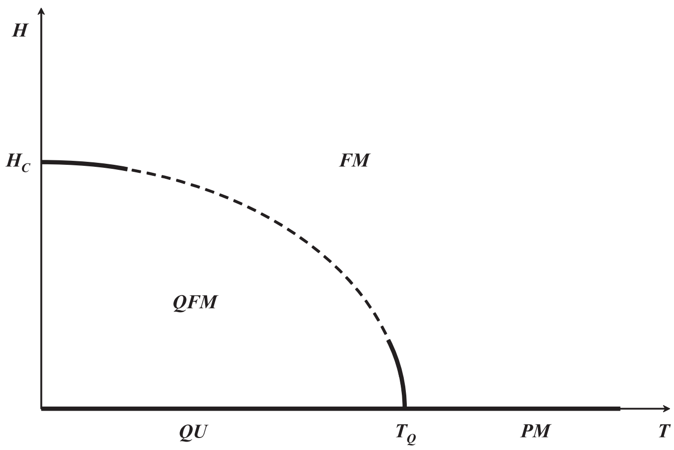

Fig. 1. The phase diagram of a non-Heisenberg 2D ferromagnet with large biquadratic exchange. FM – the ferromagnetic phase; QFM – the quadrupolar-ferromagnetic phase; QU – the quadrupolar phase; PM – paramagnetic phase; H_C – the critical field separating the FM phase and the QFM phase; T_Q – the critical temperature separating the QU phase and the PM phase.

Usually, the value of $q(0)$ is small (at $\sim T \to 0$, $q(0) \sim 0.1$), therefore it is neglected in the calculations.

As follows from (16), the integral converges in its lower limit due to the energy gap in the magnon spectrum (14). We can estimate the temperature T_Q of the QU-paramagnetic phase transition from the condition $q_2^0 = 0$.

$$T_Q \approx \frac{2\pi K_0}{3\ln\left\{\dfrac{\pi K_0}{3\sqrt{a_0\left(K_0 - J_0 + a_0\right)}}\right\}}. \qquad (17)$$

According to (17), in the absence of the ME interaction ($a_0 = 0$), $T_Q = 0$. This result is in good agreement with the Mermin-Wagner theorem, and can be explained as follows. As is obvious from the Hamiltonian (1), the parts, which describe the ME coupling through the spin structure are similar to the operators of one-ion uniaxial anisotropy. Besides, the nature of the one-ion anisotropy and the ME interaction are the same – i.e. the spin-orbit interaction (see for example [31]). Thus, the ME interaction in effect leads to the effective anisotropy which in turn leads to spontaneous symmetry breaking, and, as a consequence, to stabilization of the long-range magnetic order.

It is also necessary to note that the structure of equation (17) corresponds to the results of [32] (see Eq. (34)), in which the stabilization of the long-range magnetic order in a 2D anisotropic Heisenberg ferromagnet was investigated.

Schematically, the phase diagram of a 2D non-Heisenberg ferromagnet in an external magnetic field of the system in study is given in Figure 1. This phase diagram is plotted schematically since we have only studied the vicinity of the points ($T = 0, H_c$) and ($T_Q, H = 0$). We have approximated the rest of the phase diagram (plotted as a dashed line in Fig. 1). Actually, this phase diagram has a more complicated structure since there ought to exist other phase transitions lines, e.g. the line dividing the FM and the paramagnetic phase, which is implemented at $T > T_Q$ and $H = 0$. Therefore, this phase diagram is a rather schematic one. Nevertheless, it shows the tendency of implementing the quadrupolar phase. A more detailed study of this diagram requires further investigations, which can be the subject of future researches.

Preceding investigations [29] have shown that long-range magnetic order is absent in a 2D isotropic non-Heisenberg ferromagnet. However, as we have shown in this work, taking account even weak interactions (for example the ME coupling) leads to stabilization of not only the ferromagnetic, but also the quadrupolar order. As it was pointed out in the “Introduction”, the stabilization of long-range magnetic ordering is caused by the presence of a ME gap in the spectrum of quasimagnons. This gap is determined by spontaneous deformations (see Eq. (5)) and is static in origin. The presence of the gap in the spectrum of quasimagnons results in convergence of the fluctuation integrals on the lower limit, and, therefore, to the appearance of long-range magnetic order. It is necessary to note that in a 2D ferromagnet the temperature of phase transition from QU to paramagnetic is mainly determined by the parameters of the biquadratic exchange and the ME interaction. In the paramagnetic phase the order parameters are zero, and consequently: $\langle (S^z)^2 \rangle = \langle (S^x)^2 \rangle = \langle (S^y)^2 \rangle = \frac{2}{3}$.

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References

1. E.L. Nagaev, Usp. Fiz. Nauk **136**, 61 (1982) [Sov. Phys. Usp. **25**, 31 (1982)]
2. E.L. Nagaev, *Magnets with composite exchange interactions* (Nauka, Moskow, 1988) (in Russian)
3. Yu.N. Mitsay, A.N. Mayorova, Yu.A. Fridman, Fiz. Tverd. Tela (Leningrad) **34**, 66 (1992)
4. V.V. Val'kov, G.N. Matsuleva, S.G. Ovchinnikov, Fiz. Tverd. Tela (Leningrad) **31**, 60 (1989) [Sov. Phys. Solid State **31**, 948 (1989)]
5. Yu.N. Mitsay, Yu.A. Fridman, O.V. Kozhemyako, O.A. Kosmachev, Fiz. Nizk. Temp. **25**, 690 (1999) [Low Temp. Phys. **27**, 513 (1999)]
6. H.H. Chen, P.M. Levy, Phys. Rev. Lett. **27**, 1383 (1971)
7. H.H. Chen, P.M. Levy, Phys. Rev. **7**, 4267 (1973)
8. V.V. Val'kov, B.V. Fedoseev, Fiz. Tverd. Tela (Leningrad) **32**, 3522 (1990)
9. R. Aleonard, P. Morin, Phys. Rev. B **19**, 3868 (1979)
10. P. Morin, J. Rouchy, D. Schmitt, Phys. Rev. B **17**, 3684 (1978)
11. U. Köbler, R. Mueller, L. Smardz, D. Maier, K. Fischer, B. Olefs, W. Zinn, Z. Phys. B **100**, 497 (1996)
12. Yu.B. Gajdidej, V.M. Loktev, Fiz. Nizk. Temp. **3**, 507 (1977)

13. V.M. Kalita, Fiz. Tverd. Tela (Leningrad) **33**, 1940 (1991)
14. V.M. Kalita, A.Ph. Lozenko, Fiz. Nizk. Temp. **24**, 958 (1998)
15. A. Tanaka, T. Idogaki, Phys. Rev. B **56**, 10774 (1997)
16. K. Harada, Phys. Rev. B **65**, 052303 (2002)
17. H.A. Brown, Phys. Rev B **4**, 115 (1971)
18. A. Radomcka, T. Balcerzak, Phys. Status Solidi (a) **196**, 53 (2003)
19. U. Köbler, A. Hoser, J. Englich, A. Snezhko, M. Kawakami, M. Beyes, K. Fischer, J. Phys. Soc. Jpn **70**, 3089 (2001)
20. N. Mermin, H. Wagner, Phys. Rev. Lett. **17**, 1133 (1966)
21. S.V. Maleev, Sov. Phys. JETP **70**, 2374 (1976)
22. B.A. Ivanov, A.V. Tartakovskaya, Sov. Phys. Lett JETP **63**, 835 (1996)
23. Yu.A. Fridman, D.V. Spirin, C.N. Alexeyev, J. Magn. Magn. Mater. **234**, 174 (2001)
24. J. Hubbard, Proc. Roy. Soc. A **276**, 238 (1963)
25. J. Hubbard, Proc. Roy. Soc. A **277**, 237 (1964)
26. J. Hubbard, J. Proc. Roy. Soc. A **281**, 401 (1964)
27. R.O. Zaitsev, Zh. Eksp. Teor. Phys. **68**, 207 (1975) [Sov. Phys. JETP **41**, 100 (1975)]
28. Yu.A. Izumov, Yu.N. Skryabin, *Statistical Mechanics of Magnetically Ordered Systems* (Nauka, Moscow, 1987) (in Russian)
29. Yu.A. Fridman, D.V. Spirin, Fiz. Nizk. Temp. **26**, 374 (2000) [Low Temp. Phys. **26**, 273 (2000)]
30. Yu.A. Fridman, D.V. Spirin, Phys. Status Solidi (b) **225**, 163 (2001)
31. A.I. Achieser, V.G. Bar'yachtar, S.V. Peletminskii, *Spin Waves* (Nauka, Moskow, 1967) (in Russian)
32. S.B. Hohlachev, Sov. Phys. Lett. JETP **70**, 265 (1976)